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$$=\frac{4m^2}{\sqrt{(m^2+c^2)}}\int_0^{\frac{1}{2}\pi}\left[1+\frac{C^2\sin^2\phi}{4}+\frac{5C^4\sin^4\phi}{32}+\frac{15C^6\sin^6\phi}{128}+\text{etc.}\right]d\phi$$

$$=2\pi\sqrt{\left(\frac{m^4}{m^2+c^2}\right)}\left[1+\frac{C^2}{8}+\frac{15C^4}{256}+\frac{75C^6}{2048}+\text{etc.}\right]\ldots.(5),$$ in which $C^2=4m^2c^2/(m^2+c^2)^2$. When $c=2$ and $m^4=25$, we deduce from (5) the numerical result, $P=12.7329+$, expressing the required perimeter. With respect to the *locus* represented by (3), we have the following hypotheses: $m>c\ldots.(\alpha)$, $m=c\ldots.(\beta)$, and $m<c\ldots.(\gamma)$. Under the first hypothesis, the said locus represents the *Cassinian Oval*; under the second hypothesis, the said locus represents the *Bernoullian Lemniscate*; and under the third hypothesis, the said locus represents two *ovaliform figures*.

In rectifying under the hypotheses (β) and (γ), the term $(m^2-c^2)^2$ in (4) and in (g) must be altered accordingly.

[To be continued.]

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THE CONSTRUCTION OF THE SUN'S PATH.

By ERIC DOOLITTLE, Professor of Mathematics in the State University of Iowa, Iowa City, Iowa.

In the *Archiv der Mathematik and Physik*, Vol. LIII. Part IV., there is a very interesting article by Professor Hoza on the graphical construction of the sun's apparent path. He considers the earth as stationary in its orbit during a perod of twenty-four hours, and obtains the projection of the apparent path during that time upon the plane of the Equator; the result being, as might be expected, an ellipse, the ratio of whose axes is as 1 : cos δ.

This construction only applies to those places on the earth where the sun actually rises and sets each day, nor is the exact path thus found, since the sun's motion is not taken into account. An investigation of this latter is not difficult; it will lead us to a very interesting spiral curve.

Let us take the vernal equinox as an origin; the arc of the equator as an axis of X positive toward the right, and a great circle perpendicular to the equator through the vernal equinox as the axis of Y. The circles lie on the celestial sphere, whose radius is considered as unity.

Then, if c be the angular velocity of the earth on its axis, and K that of the sun in the ecliptic, (considered as uniform), we will have at a time t after the time of vernal equinox:

$$x=\tan^{-1}\{\cos e\tan Kt\}-ct\ldots\ldots(1)$$

$$y=\sin^{-1}\{\sin e\sin Kt\}\ldots\ldots(2)$$ where e, the obliquity of the ecliptic, is the constant angle 23° 27′.

Also, consider the point (x, y) as orthogonally projected upon the plane of the equator, and let the radius vector from the center of the sphere to the projected point be ρ. We may write the additional equation,

$$\rho^2=\cos^2 y=(1-\sin^2 e\sin^2 Kt)\ldots\ldots(3).$$

By eliminating t between (1) and (2) we may obtain the equation of the path in spherial co-ordinates, or with (3) and the formula

$$\rho\sin\varphi=\sin x\ldots\ldots(4),$$ we may find the equation of the projection upon the equator in polar co-ordinates. It will be easiest to examine the path by the help of (1), (2), and (3) without eliminating.

By differentiating (1)

$$\frac{dx}{dt}=\frac{K\cos e}{\sin^2 e\sin^2 Kt+\cos^2 e}-c.$$

The maximum value of the first term of this expression occurs when $t=0$: it is $K\sec e$ or, about $1.1K$. But $c=366\frac{1}{4}K\ldots\ldots(5)$ and hence $\frac{dx}{dt}$ is always minus, and x is always a decreasing function of t.

Similarly, $\frac{dy}{dt}=K\sin e\frac{\cos Kt}{\sqrt{1-\sin^2 Kt\sin^2 e}}$ and y increases as Kt increases from 0 to $\frac{\pi}{2}$, at which point y attains the maximum value, e. But, by (5), as Kt increases to $\frac{\pi}{2}$, ct increases to $183\frac{1}{8}\pi$, and, from (1), x decreases to $-182\frac{5}{8}\pi$. Thus the spiral begins at the vernal equinox, and reaches its highest point after $91\frac{5}{16}$ revolutions about the sphere.

When Kt increases to π, y decreases to 0, and x decreases to $-365\frac{1}{4}\pi$, so that the spiral turns downward and crosses the equator after $182\frac{5}{8}$ revolutions. The negative values are similar and show that the spiral begins to repeat itself after $365\frac{1}{4}$ revolutions.

Equation (3) shows us that the projection on the plane of the equator is a similar spiral, which begins at the vernal equinox and draws continually nearer the pole for 91 revolutions, after which it widens out until $\rho=1$ at the autumnal equinox. The minumum value of ρ is $\cos e$, and the curve is always concave toward the pole.

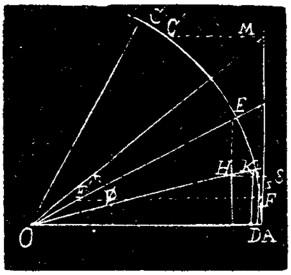
We may make an accurate construction of this projection by points, as follows:

Draw the unit circle and lay off the given angles and their sines:

$$AOK=Kt;\quad AOE=e;\quad AOC=ct.$$

Say $OF'=\sin Kt$ and draw $F'F$ and FD.

Then $AOF=y$ (by (2)) and $OD=\cos y=\rho$. Draw HS, meeting the tangent in z. Then $AOZ=$first term of x and if AZ' be laid back-



ward from C to C', $AOC'=x$. Finally, draw $C'M$ to DF; then $AOM=\varphi$ by (4), and AOM and OD are co-ordinates of points on our spiral. In the figure, AK was taken 10°, and thus $AC=20\pi+62\frac{1}{8}^\circ$.

So accurate a construction might be used sometimes; for instance for the time of sun-rise and sun-set especially near the poles, that is where the spiral cuts the projected Horizon circle. For showing the form of the spiral it can of course be much shortened: $\tan^{-1}(\cos e \tan Kt)$ may be written Kt, and in fact, when it is noticed how very nearly parallel the spires are, many convolutions may be interpolated without computation.

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A METHOD OF INTEGRATING CERTAIN DIFFERENTIAL EQUATIONS OF THE FIRST ORDER AND N-TH DEGREE.

By RALPH H. KUNSTADTER, Graduate Student, Yale College.

The regular solution of the equation $e^{ax}f(p)\pm e^{by}f'(p)=0$, (where $p=\frac{dy}{dx}$) as given in text-books on differential equations is performed after substituting x' far e^{ax} and y' for e^{by}.

In the following demonstration, I shall endeavor to give a very rational and perhaps a natural method which might be easily generalized for similar equations consisting of two members only.

Regarding our equation, we at once recognize it as being a trancendental and exponential equation, which in order to solve we will bring into logarithmic form. For this purpose, multiply $e^{ax}f(p)+e^{by}f'(p)=0\ldots.(1)$ by $e^{ax}f(p)-e^{by}f'(p)=0$ and we have $e^{2ax}f^2(p)-e^{2by}f'^2(p)=0\ldots.(2)$.

Applying logarithms, we have $\log e^{2ax}+2\log f(p)=\log e^{2by}+2\log f'(p)\ldots.(3)$. Or $ax+\log f(p)=by+\log f'(p)\ldots.(4)$.

Differentiating this equation, we have

$$adx+\frac{\varphi(p)dp}{f(p)}=bdy+\frac{\varphi'(p)dp}{f'(p)}\ldots.(5).$$

Dividing by dx and clearing of fractions,

$$f(p)f'(p)a+[\varphi(p)f'(p)-f(p)\varphi'(p)]\frac{dp}{dx}+bpf'(p)f(p)=0\ldots.(6).$$ Or

$$\frac{dp}{dx}[\varphi(p)f'(p)-f(p)\varphi'(p)]+af(p)f'(p)+bpf'(p)f(p)=0.$$